**TOPOLOGY I - BACK PAPER EXAM**

Time : 180 minutes Max. Marks : 100

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

(1) (a) Let $d, \rho$ denote the euclidean and the square metric on $\mathbb{R}^n$ respectively. Prove that

$$\rho(x, y) \leq d(x, y) \leq \sqrt{n}\, \rho(x, y)$$

for all $x, y \in \mathbb{R}^n$. Show that the topologies induced by $d$ and $\rho$ are the same as the product topology on $\mathbb{R}^n$. [3+8]

(b) Prove that $\mathbb{R} \times \mathbb{R}$ in the dictionary order topology is metrizable. [6]

(2) (a) Define the term : *quotient map.* Show that a continuous surjective map $p : X \longrightarrow Y$ between topological spaces is a quotient map if and only if $p$ maps saturated open sets to open sets. [1+6]

(b) Let $p : X \longrightarrow Y$ be a quotient map. Show that if $Y$ is connected and $p^{-1}(y)$ is connected for each $y \in Y$, then $X$ is connected. [10]

(3) Give examples of the following :

(a) A connected space with infinitely many path components.

(b) A connected space that is not locally path connected

(c) A Lindelof space $X$ with $X \times X$ not Lindelof.

(d) A Lindelof space $X$ and a subspace $Y$ of $X$ that is not Lindelof.

In each case give complete justificaions. [4x6=24]

(4) Let $X, Y$ be spaces with $Y$ compact. Show that the projection $\pi_1 : X \times Y \longrightarrow X$ to the first factor is a closed map. [10]

(5) Prove that a regular Lindelof space is normal. [13]

(6) (a) Define the term *retract.* Show that a retract of a Hausdorff space is closed. [1 + 6]

(b) Define the term *deformation retraction.* If $A$ is a deformation retract of $X$, show that $A$ is homotopically equivalent to $X$. [2+10]